<u>Part I: Calculus II Final Project</u>

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Calculus II focuses on integration, sequences, and infinite series that evolved centuries ago. The earliest ideas of Calculus II came from Archimedes and Eudoxus with their “method of exhaustion” (Bressoud, n.d.). “The method of exhaustion is the approximation of the area using ever-smaller pieces whose area can be found exactly, and then proving that a particular value is the answer by showing that anything smaller is too small and anything larger is too large” (Bressoud, n.d.). Archimedes’ studies have used the method of exhaustion to calculate approximate areas and volumes (EBSCO, 2022). Calculus II came along in the 18th century by Sir Isaac Newton and Gottfried Leibniz who independently developed the laws of differentiation and integration. Newton worked on differentiation, rates of change whereas Leibniz began with integration (Oxford Scholastica Academy). After the works of Newton and Leibniz, Leonhard Euler and Brook Taylor expanded their ideas of calculus to include powers, exponents and trigonometric series which became the heart of Calculus II (Connor & Robertson, 1996). During the 19th century, Augustin-Louis Cauchy introduced the “Cours d’analyse” which is a book of his complex analysis such as integrals and integration formulae in mathematics (Eagan, 2024). Following Cauchy, Bernhard Riemann brought out the Riemann sum which is named after him. “A Riemann sum is the method for approximating the area underneath a curve by splitting the area into rectangular sub intervals. Finding the area underneath the curve is also known as an integral” (Kim, n.d.). Riemann’s sum was not perfect enough so French mathematician Henri Lebesgue formulated new integral concepts to address issues with Riemann’s sum (Barnett, 2016). His work modified the integral concept and shaped modern mathematical analysis (Barnett, 2016). Today, Calculus II bridges the gap between mathematics and modern analysis, carrying on the legacy of integration and infinite series.

Mathematical modeling builds connections between theoretical mathematical models and real-world problem solving processes. It “refers to the process of creating a mathematical representation of a real-world scenario to make a prediction or provide insight” (Society for Industrial and Applied Mathematics (SIAM, n.d.). This process starts from defining the problem. It requires “modelers should clearly specify what it is they would like to find” (SIAM, n.d.). Since “it is impossible to account for all the factors that impact a given situation… the modeler must choose what is most important to include” (SIAM, n.d.). In other words, modelers decide on what they focus on the most, thereby establishing key factors that lead to the situation. Based on the assumptions about the factors that drive the situation, which help reveal “variables that will be needed” (SIAM, n.d.), modelers “start with a small number of variables representing only the most important factors” (SIAM, n.d.). After setting up both independent and dependent variables within the relationship, modelers “use familiar mathematical and computational tools to solve” (SIAM, n.d.). “When considering the results and insights gained from the model” (SIAM, n.d.), modelers ask themselves “if the answer makes sense” (SIAM, n.d.). Moreover, modelers “refine the model by repeating the process, adjusting as needed to improve the solution” (SIAM, n.d.). As the report on “the model, solution technique, and results” (SIAM, n.d.) is produced, modelers engage in further discussion with the research group.

Building upon the general description of mathematical modeling and its main components, “the exploration of mathematical modeling through differential equations unveils a powerful tool for understanding and predicting the behavior of dynamic systems in various fields” (Patil, 2023, p. 4). According to Patil (2023), differential equations “capture how a system evolves over time or in response to changing conditions … For instance, they can describe the growth of populations, the spread of diseases, the motion of celestial bodies, and the behavior of fluids" (p. 1). To put it differently, differential equations describe a system where future states rely on current states, particularly modeling population dynamics and revealing natural patterns. In addition to differential equations, another key concept in Calculus II is integration. It "entails adding up a large number of very little amounts (and then reaching a limit as the size of the quantities approaches zero while the number of terms approaches infinity) … Definite integrals are used to calculate area, volume, gravity center, moment of inertia, force-driven work, and a variety of other applications" (Unacademy, n.d.). This is to say, within the framework of integration, quantities are accumulated through infinite subtle additions. Drawing on the concept of limit, as infinitely adding infinitely subtle changes, the accumulated quantities approach a definite number if given the range. Therefore, integration is essentially important in the field of adding up each unit in order to calculate accumulation, such as engineering and physics.

Social Justice in calculus refers to the idea that all students deserve equal opportunity, fair treatment, and access to resources for a better education. This includes students of every gender, race, level of income, housing situation, and community background. According to the article "Teaching Math Through a Social Justice Lens," teachers are increasingly using real-life issues like food deserts, policing patterns, and health inequality to help students see math as a tool to understand the world (Education Week). On the website "Math and Social Justice" by RadicalMath, it is pointed out that math can help examine issues like environmental racism, housing discrimination, and economic exploitation (RadicalMath). In the context of a Calculus II class, this means we can use integrals, rates of change, and mathematical models to measure and analyze how inequality develops over time, how resources are distributed, or how environmental hazards impact different communities. This kind of approach helps students not only to learn calculus skills but to see how those skills can be used to support fair decision-making and positive social change.

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